

# X(3872) as a hybrid state of the charmonium and the hadronic molecule

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In order to understand the structure of the  $X(3872)$ , the  $c\bar{c}$  charmonium core state coupling to the  $D^0\bar{D}^{*0}$  and  $D^+D^{*-}$  molecular states are studied. The strengths of the couplings between the charmonium state and the hadronic molecular states are determined so as to reproduce the observed mass of the  $X(3872)$ . The  $D\bar{D}^*$  attraction is taken to be consistent with the observed  $Z_b(10610)$  and  $Z_b(10650)$  masses. The isospin symmetry breaking is introduced by the mass differences of the neutral and the charged  $D$  mesons. The obtained structure of the  $X(3872)$  is that about 6% is  $c\bar{c}$  charmonium, 69% is the isoscalar  $D\bar{D}^*$  molecule and 26% is the isovector  $D\bar{D}^*$  molecule, which explains many of the observed properties of the  $X(3872)$ , such as the isospin symmetry breaking, the production rate in the  $p\bar{p}$  collision, a lack of the existence of the  $\chi_{c1}(2P)$  peak predicted by the quark model, and the absence of the charged  $X$ . The structure of the  $X(3872)$  we have obtained is not just a  $D^0\bar{D}^{*0}$  hadronic molecule but the charmonium-hadronic molecule hybrid state.

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## I. INTRODUCTION

The  $X(3872)$  state was first observed in 2003 by Belle in  $B^\pm \rightarrow J/\psi \pi^+ \pi^- K^\pm$  [1] and was confirmed by CDF [2], D0 [3] and *BABAR* [4] collaborations. The observed masses of the  $X(3872)$  in the  $J/\psi \pi^+ \pi^-$  channel from the recent measurements of the charged and the neutral  $B$  decays are  $(3871.4 \pm 0.6 \pm 0.1)$  MeV and  $(3868.7 \pm 1.5 \pm 0.4)$  MeV, respectively [5]. Those from the  $p\bar{p}$  and the  $pp$  collisions are  $(3871.61 \pm 0.16 \pm 0.19)$  MeV [6] and  $(3871.95 \pm 0.48 \pm 0.12)$  MeV [7]. The average mass given by the particle data group in 2012 [8] is  $(3871.68 \pm 0.17)$  MeV, which is 0.16 MeV below the  $D^0\bar{D}^{*0}$  threshold. The full width is less than 1.2 MeV.

As for the spin-parity quantum numbers of the  $X(3872)$ , the angular distributions and correlations of the  $\pi^+\pi^-J/\psi$  final state have been studied by CDF [9] and they concluded that the pion pairs originate from  $\rho^0$  mesons and that the favored quantum numbers of the  $X(3872)$  are  $J^{PC} = 1^{++}$  and  $2^{-+}$ . Recent analyses support  $J^{PC} = 1^{++}$  interpretation [10]. *BABAR* has found the evidence of the radiative decays of  $X(3872) \rightarrow \gamma J/\psi$  with 3.4-3.6  $\sigma$  significance [11, 12], which implies that the  $C$ -parity of  $X(3872)$  is positive. Though we assume that  $X(3872)$  is  $J^{PC} = 1^{++}$ , whether the quantum number is  $1^{++}$  or  $2^{-+}$  is still an issue of the discussion and more experimental data are certainly necessary.

Since the first observation of the  $X(3872)$ , it has received much attention because its features are difficult to explain if a simple  $c\bar{c}$  bound state of the quark potential model is assumed [13].  $X(3872)$  is one of the

promising candidates of the exotic states reviewed in Ref. [14–16]. Many kinds of structures have been suggested for the  $X(3872)$  from the theoretical side, such as a tetraquark structure [17–20], a  $D^0\bar{D}^{*0}$  molecule [21–28] and a charmonium-molecule hybrid [29–31]. We also employ this hybrid picture and argue that that is most appropriate.

One of the important properties of the  $X(3872)$  is its isospin structure. The branching fractions measured by Belle [32] is

$$\frac{Br(X \rightarrow \pi^+\pi^-\pi^0 J/\psi)}{Br(X \rightarrow \pi^+\pi^- J/\psi)} = 1.0 \pm 0.4 \pm 0.3, \quad (1)$$

and  $(0.8 \pm 0.3)$  by *BABAR* [33]. Here the two-pion mode originates from the isovector  $\rho$  meson while the three-pion mode comes from the isoscalar  $\omega$  meson. So, the eq. (1) indicates strong isospin violation. M. Suzuki has estimated the kinematical suppression factor including the difference of the vector meson decay width and obtained the production amplitude ratio [34] using Belle's value

$$\left| \frac{A(\rho J/\psi)}{A(\omega J/\psi)} \right| = 0.27 \pm 0.02. \quad (2)$$

Usual size of the isospin symmetry breaking is at most a few %. It is interesting to know what is the origin of this strong isospin symmetry breaking. In [35], this problem has been studied by using the chiral unitary model and the effect of the  $\rho$ - $\omega$  mixing has been discussed in [36]. It was reported that both of the approaches can explain the observed ratio given in Eq. (1), but at present, consensus on the mechanism of the large isospin symmetry breaking has yet to be reached. We will show in this work that the mass difference of the  $D^0\bar{D}^{*0}$  and the  $D^+D^{*-}$  thresholds gives enough amount of the isospin violation to explain the experiments.

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The hadronic decays of the  $X(3872)$  are investigated in [37–45] and the radiative decays in [46–54], which depend on rather detailed of the wave function. They are beyond the scope of this paper and we will discuss the matter elsewhere. Let us mention briefly, however, that the production processes have been studied in [55–58], and it seems that a pure molecule picture cannot explain the production rate of the  $X(3872)$  in the  $p\bar{p}$  collision well [57]. There, the production rate of the  $X(3872)$  is about 1/20 of the rate of  $\psi(2S)$ , which suggests that  $X(3872)$  has to have, very roughly speaking, the order of 5% of the  $c\bar{c}$  component.

Recently,  $Z_b(10610)^{\pm,0}$  and  $Z_b(10650)^{\pm,0}$  ( $J^P=1^+$ ) resonances have been found in the  $\Upsilon(5S)$  decay to  $\Upsilon(nS)\pi^+\pi^-$  ( $n=1,2,3$ ) and  $h_b(mP)\pi^+\pi^-$  ( $m=1,2$ ) reactions[59]. The masses of these resonances are just above to the  $BB^*$  and the  $B^*B^*$  thresholds, respectively; the main components are considered to be the  $B^{(*)}B^*$  two-meson states. This means that there exist almost-zero-energy bound states (or resonances) in both of the  $D\bar{D}^*$  and the  $B^{(*)}\bar{B}^*$  systems. In order to make such states, the attraction in the  $D\bar{D}^*$  system is considered to be about 2.7 times as strong as that of the  $B^{(*)}\bar{B}^*$  system because the reduced mass of the  $D\bar{D}^*$  system, 967MeV, is about 2.7 times as light as that of the  $B^{(*)}\bar{B}^*$  system, 2651MeV. On the other hand, the interaction between the  $D$  and  $\bar{D}^*$  mesons is probably about the same size as that between the  $B^{(*)}$  and  $\bar{B}^*$  mesons. We argue that the extra attraction required for the  $X(3872)$  comes, at least mainly, from its coupling to the  $c\bar{c}$  core, which is absent in the  $Z_b$  systems.

It is also an important issue that whether the charged partner of the  $X(3872)$  exists as a measurable peak or not. *BABAR* has searched such a state in the  $X \rightarrow \pi^-\pi^0 J/\psi$  channel and found no signal [60]. The above mechanism, where the coupling to the  $c\bar{c}$  core plays a major role to bound the neutral  $X$ , is consistent with the absence of the charged  $X$ .

The  $X(3872)$  exists above the open charm threshold,  $D\bar{D}$ . Below this threshold, the  $c\bar{c}$  mass spectrum is well predicted by a simple quark model. The model, however, failed to predict the masses above the open charm threshold, especially for the  $D\bar{D}$   $S$ -wave sector. In this work, we also look into the  $c\bar{c}$  to  $D\bar{D}^*$  transfer strength and clarify that the  $c\bar{c}$  peak can actually disappear by the  $c\bar{c}$ - $D\bar{D}^*$  coupling.

In this article, we present a hybrid picture where  $X(3872)$  is  $J^{PC} = 1^{++}$  and consists of  $D^0\bar{D}^{*0}$ ,  $D^+\bar{D}^{*-}$ , and the  $2^3P_1$   $c\bar{c}$  core, which stands for the  $\chi_{c1}(2P)$  if observed. A separable  $D\bar{D}^*$  interaction is introduced, whose strength is determined so as to give a zero-energy bound state when it applied to the  $B^{(*)}\bar{B}^*$  systems. The rest of the required attraction to form the  $X(3872)$  are assumed to come from the  $c\bar{c}$ - $D\bar{D}^*$  coupling. The coupling strength is determined so as to give the observed  $X(3872)$  mass. The  $c\bar{c}$  core mass is taken from the quark model result, and the cutoff is chosen by considering the  $c\bar{c}$  core size. As we will discuss later, the behaviors of the

$X(3872)$  do not depend strongly on the detail of the interactions. Main parameters of the present model are the overall strength of the two-meson interaction and that of the coupling, which are essentially determined from the masses of  $X(3872)$  and  $Z_b$ 's. This simple picture, however, is found to be consistent with many of the experiments, such as the isospin symmetry breaking in the  $X(3872)$  decay, the production rate of  $X(3872)$  in the  $p\bar{p}$  collision, and the absence of  $\chi_{c1}(2P)$  peak or the charged  $X$ , in addition to the mass of  $X(3872)$  and  $Z_b$ 's, which are the inputs.

Let us briefly mention features of our work in comparison to those that also employ the charmonium-molecule hybrid model [29–31]. In Ref. [29], the hybrid structure of the  $X(3872)$  has been studied in the QCD sum rule approach by considering a mixed charmonium-molecular current. They found a very deeply bound  $X(3872)$ ,  $m_X = (3.77 \pm 0.18)$  GeV, 97% of whose component is a charmonium. As we shall show in Sec.II, the structure of the  $X(3872)$  certainly depends on the binding energy strongly. The effective hadronic models with the charmonium- $D\bar{D}^*$  molecule transition interaction have been used in Ref. [30, 31] as well as the present work. Danilkin and Simonov [30] have studied the  $D\bar{D}^*$  production spectrum. They obtained the strength of the  $c\bar{c}$ - $D\bar{D}^*$  coupling from the heavy quarkonium decay calculated by a quark model with a small adjustment. They certainly found a steep rise near the threshold. We have studied the  $B \rightarrow X(3872) K$  or  $D\bar{D}^* K$  weak decay spectrum at almost same time independently in the very similar approach in [61, 62]. Here, we also introduce the interaction between  $D$  and  $\bar{D}^*$ , and look into the features of the shallowly bound  $X(3872)$ . The work in Ref. [31] they examined the effects of the Okubo-Zweig-Iizuka forbidden  $\rho^0 J/\psi$  and  $\omega J/\psi$  channels. Since the main decay modes of the  $X(3872)$  are the  $X(3872) \rightarrow \pi^+\pi^- J/\psi$  and  $X(3872) \rightarrow \pi^+\pi^-\pi^0 J/\psi$ , their inclusion is certainly important. As shown in [31], however, this effect on the pole position seems rather small. To avoid the complication we discuss it elsewhere. In the present study, we have introduced the attractive interaction between  $D$  and  $\bar{D}^*$  mesons with the coupling strength being consistent with the observed  $Z_b(10610)$  and  $Z_b(10650)$  masses. This point is new to the previous two studies and we consider that we can successfully draw the consistent picture of the observed exotic hadrons  $X(3872)$ ,  $Z_b(10610)$  and  $Z_b(10650)$ .

One of the authors (S. T.) has studied the  $X(3872)$  using a quark potential model by introducing an extra ( $q\bar{q}$ ) pair to a  $c\bar{c}$  system [63] and found a shallow bound state of  $q\bar{q}c\bar{c}$  with  $J^{PC} = 1^{++}$ . Recently, an elaborate study has been done in the quark potential model [64]. They have performed the coupled channel calculations including two and four-quark configurations using the  $^3P_0$  model and found a good agreement with the experimental data. The purpose of the present work is to make the situation of the  $X(3872)$  clearer by studying the role of the  $c\bar{c}$  core state, which couples to the  $D^0\bar{D}^{*0}$  and

$D^+D^{*-}$  molecular states, with a simple hadronic model. This approach will complement the picture given by the quark model approach.

This paper is organized as follows. In Sec.II, the calculation of the  $X(3872)$  state is given. In Sec.III, we discuss the transition strength of the weak decay of B meson:  $B \rightarrow X(3872)K$  or  $D\bar{D}^*K$  using the Green's function approach. In Sec.IV, we study the effects of the interaction between the  $D$  and  $\bar{D}^*$  mesons. Finally, Sec.V is devoted to summary of this paper.

## II. $X(3872)$

We argue that the  $X(3872)$  state is a superposition of the  $c\bar{c}$  core state, the  $D^0\bar{D}^{*0}$  hadronic molecular state, and the  $D^+D^{*-}$  hadronic molecular states. So, the wave function of the  $X(3872)$  in the center of the mass frame is represented by

$$|X\rangle = c_1 |c\bar{c}\rangle + c_2 |D^0\bar{D}^{*0}\rangle + c_3 |D^+D^{*-}\rangle. \quad (3)$$

The  $D^0\bar{D}^{*0}$  and  $D^+D^{*-}$  molecular states are given by

$$|D^0\bar{D}^{*0}\rangle = \int d^3\mathbf{q} \varphi_0(\mathbf{q}) |D^0\bar{D}^{*0}(\mathbf{q})\rangle, \quad (4)$$

$$|D^+D^{*-}\rangle = \int d^3\mathbf{q} \varphi_+(\mathbf{q}) |D^+D^{*-}(\mathbf{q})\rangle, \quad (5)$$

where  $\mathbf{q}$  represents the relative momentum of the  $D$  and  $\bar{D}^*$  mesons. The normalization of the states are

$$\begin{aligned} \langle D^0\bar{D}^{*0}(\mathbf{q}') | D^0\bar{D}^{*0}(\mathbf{q}) \rangle &= \langle D^+D^{*-}(\mathbf{q}') | D^+D^{*-}(\mathbf{q}) \rangle \\ &= \delta^3(\mathbf{q}' - \mathbf{q}). \end{aligned} \quad (6)$$

Here  $\varphi_0(\mathbf{q})$  and  $\varphi_+(\mathbf{q})$  are the momentum representation of the wave functions of the  $D^0\bar{D}^{*0}$  and  $D^+D^{*-}$  hadronic molecular states, respectively. The charge conjugation is assumed to be positive throughout this paper.

We introduce a coupling between the  $c\bar{c}$  core state and the  $D\bar{D}^*$  states in the isospin symmetric manner. Since we are looking into the low energy region, the results do not depend much on the shape of the interaction. Thus, we take a monopole-type coupling as:

$$\begin{aligned} \langle D^0\bar{D}^{*0}(\mathbf{q}) | V | c\bar{c} \rangle &= \langle D^+D^{*-}(\mathbf{q}) | V | c\bar{c} \rangle \\ &= \frac{g}{\sqrt{\Lambda}} \left( \frac{\Lambda^2}{q^2 + \Lambda^2} \right). \end{aligned} \quad (7)$$

The interaction we have introduced above causes effectively an attraction for the  $X(3872)$  because its energy is lower than the mass of the  $c\bar{c}$  core,  $m_{c\bar{c}}$ . In this section, we ignore the direct interactions between the  $D$  and  $\bar{D}^*$  mesons; as we will discuss later in Sec.IV, the coupling to the  $c\bar{c}$  core seems more important to make the  $X(3872)$  than the direct  $D\bar{D}^*$  attraction.

Here we consider only the relative  $S$ -wave states of these two mesons in the non-relativistic scheme because the  $X(3872)$  is close to the threshold. The Schrödinger equation to solve is

$$\begin{pmatrix} m_{c\bar{c}} - E & V & V \\ V & m_{D^0} + m_{D^{*0}} + \frac{\vec{p}^2}{2\mu_0} - E & 0 \\ V & 0 & m_{D^+} + m_{D^{*-}} + \frac{\vec{p}^2}{2\mu_+} - E \end{pmatrix} \begin{pmatrix} c_1 |c\bar{c}\rangle \\ c_2 |D^0\bar{D}^{*0}\rangle \\ c_3 |D^+D^{*-}\rangle \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad (8)$$

with

$$\frac{1}{\mu_0} = \frac{1}{m_{D^0}} + \frac{1}{m_{D^{*0}}}, \quad \frac{1}{\mu_+} = \frac{1}{m_{D^+}} + \frac{1}{m_{D^{*-}}}. \quad (9)$$

Since the interaction we employ is separable, we can solve this Schrödinger equation analytically. The bound state energy is obtained by solving the following equation.

$$m_{c\bar{c}} - E - F_0(E) - F_+(E) = 0, \quad (10)$$

with

$$F_0(E) = \int \frac{d^3\mathbf{q}}{\left(m_{D^0} + m_{D^{*0}} + \frac{q^2}{2\mu_0}\right) - E} \left( \frac{g\Lambda^{3/2}}{q^2 + \Lambda^2} \right)^2, \quad (11)$$

and

$$F_+(E) = \int \frac{d^3\mathbf{q}}{\left(m_{D^+} + m_{D^{*-}} + \frac{q^2}{2\mu_+}\right) - E} \left( \frac{g\Lambda^{3/2}}{q^2 + \Lambda^2} \right)^2. \quad (12)$$

For later convenience, we define  $\alpha_0$  and  $\alpha_+$  by

$$\frac{\alpha_0^2}{2\mu_0} = m_{D^0} + m_{D^{*0}} - m_X, \quad (13)$$

and

$$\frac{\alpha_+^2}{2\mu_+} = m_{D^+} + m_{D^{*-}} - m_X, \quad (14)$$

where  $m_X$  represents the observed mass of the  $X(3872)$ .

In order to obtain the numerical results, we use the  $D$  meson masses given in the 2012 Review of Particle Physics [8] (Table I). Since we have introduced the isospin symmetric interaction  $V$  in Eq. (7), the only origin of the isospin violation in the present model is the mass difference between the charged and neutral  $D$  and  $D^*$  mesons.

As for the  $c\bar{c}$  core state, we consider that it corresponds to the  $J^{PC} = 1^{++}$  charmonium state with the mass  $m_{c\bar{c}} = 3.950$  GeV, the closest  $c\bar{c}$  core to  $X$ . This value is taken from the Godfrey and Isgur's results of the quark potential model calculation for the  $2^3P_1$   $c\bar{c}$  state [65].

In the following, we will show the results as well as their dependence on the various assumptions.

There are two free parameters in the present model: the cutoff  $\Lambda$  and the dimensionless coupling constant  $g$ . We take typical hadron sizes for  $\Lambda$ : *e.g.*,  $\Lambda = 0.3$  GeV,  $0.5$  GeV and  $1.0$  GeV. Then, for a given  $\Lambda$ , the coupling constant  $g$  is determined so that the model reproduces the observed mass of the  $X(3872)$ , namely,  $3.87168$  GeV. The results are given in Table II. The wave function we have obtained:

$$\begin{aligned} |X\rangle &= c_1 |c\bar{c}\rangle + c_2 |D^0 \bar{D}^{*0}\rangle + c_3 |D^+ D^{*-}\rangle \\ &= c_1 |c\bar{c}\rangle + c_{I=0} |D\bar{D}^*; I=0\rangle + c_{I=1} |D\bar{D}^*; I=1\rangle. \end{aligned} \quad (15)$$

The values of  $c$ 's are shown in Table III for each of the cutoff values. It seems that the overall feature of the admixture of each component does not depend much on the value of  $\Lambda$ , which is not surprising because a very shallow state does not depend much on detail of the potential. The main component of the  $X(3872)$  state is  $|D^0 \bar{D}^{*0}\rangle$ , reflecting the fact that the mass of the  $X(3872)$  is only  $0.16$  MeV below the  $D^0 \bar{D}^{*0}$  threshold. The amplitude of the  $|D^+ D^{*-}\rangle$  component is much smaller because the  $D^+ D^{*-}$  threshold is  $8.22$  MeV above the mass of the  $X(3872)$ . The size of the isospin symmetry breaking we have obtained for the averaged mass,  $3.87168$  GeV, seems to be roughly consistent with the one estimated from the experiments given by Eq. (2).

Let us emphasize that we have obtained a sizable amount of the  $|c\bar{c}\rangle$  component. In the present scheme, the origin of the attraction is the coupling between the  $D\bar{D}^*$  component and  $c\bar{c}$  core state. So, it is natural to have a rather large  $|c\bar{c}\rangle$  component in the  $X(3872)$  state.

The mass of the  $c\bar{c}$  core is taken from the quark model calculation. It may differ because of the model assumption. The slight change of its mass, however, does not

change our results much. For example, for  $m_{c\bar{c}} = 3.93$  or  $3.97$  GeV, which we take because  $\pm 20$  MeV is typical ambiguities of the quark models, the  $c\bar{c}$  component becomes  $11\%$  and  $7\%$ , respectively. There is no drastic change in the results.

We take only this state as the  $c\bar{c}$  core state in this paper because it has the closest mass to that of  $X(3872)$ . The mass of the  $1^3P_1$   $c\bar{c}$  state is, for example, around  $3.5$  GeV and therefore its coupling to the  $X(3872)$  will be suppressed [64]. In our calculation which includes the lower  $c\bar{c}$  state with the same coupling size, the probability of the  $1^3P_1$   $c\bar{c}$  state is found to be about  $1/20$  of that of the  $2^3P_1$   $c\bar{c}$  state. The existence of another core component may change the nature of the  $\gamma$ -decay of  $X(3872)$ , where a large cancellation occurs and results are very sensitive to the wave function [66, 67]. We, however, look into such observables elsewhere, and concentrate on the bulk feature of  $X(3872)$  in this work.

Experimental uncertainty of the  $X(3872)$  mass still exists. So, we solve the system also for  $m_X = 3.8687$  GeV. This mass is the one determined from the neutral  $B$  meson decay data, and the lightest mass among the ones given by the experiments. Now the binding energy becomes  $3.14$  MeV instead of the one corresponding to the average mass,  $0.16$  MeV. The value of  $g$  to form the lighter mass becomes  $0.05625$ , which is  $1.1$  times as large as that of the average mass,  $0.05110$ . In order to form the more deeply bound  $X$ , the dimensionless coupling constant  $g$  is required to be larger. The coefficients of the wave function are listed in Table III. The size of the  $c\bar{c}$ -core component also becomes larger: it changes from  $(0.293)^2 \simeq 0.086$  to  $(0.522)^2 \simeq 0.272$  as  $m_X$  changes from  $3.87168$  to  $3.8687$  GeV for the case of  $\Lambda = 0.5$  GeV. The size of the  $c\bar{c}$ -core component in the  $X(3872)$  is found to be sensitive to the binding energy of the state. The amount of the isospin symmetry breaking depends also on the binding energy of  $X$ ; The symmetry breaking occurs because of the difference of the binding energies of the  $X(3872)$  from the two thresholds, *i.e.*,  $D^0 \bar{D}^{*0}$  and  $D^+ D^{*-}$ . For  $m_X = 3.87168$  and  $3.8687$  GeV, the ratios of the size of the isovector to the isoscalar  $D\bar{D}^*$  components are  $0.315$  and  $0.057$ , respectively. When the mass of the  $X(3872)$  becomes smaller, namely, the binding energy becomes larger, the effects of the threshold difference becomes smaller, and the isospin violation becomes smaller.

Let us show the shape of the obtained wave functions. The explicit expressions of the wave functions in the co-

TABLE I: Meson masses and the thresholds. All the entries are in GeV.

$m_{D^0}$	$m_{D^+}$	$m_{D^{*0}}$	$m_{D^{*-}}$	$m_{D^0} + m_{D^{*0}}$	$m_{D^+} + m_{D^{*-}}$
1.86486	1.86962	2.00698	2.01028	3.87184	3.87990

TABLE II: The values of the dimensionless coupling constant  $g$  for each value of the cutoff  $\Lambda$  in units of GeV. The mass of  $X(3872)$  is  $m_X = 3.87168$  GeV.

$\Lambda$ [GeV]	0.3	0.5	1.0
$g$	0.05435	0.05110	0.04835

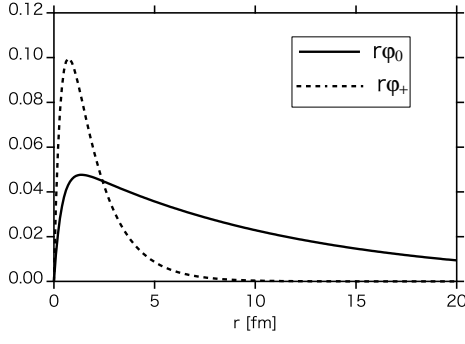


FIG. 1: The  $D\bar{D}^*$  components of the  $X(3872)$  wave function for the  $m_X = 3.87168$  GeV and  $\Lambda = 0.5$  GeV case.  $D^0\bar{D}^{*0}$  wave function,  $r\varphi(r)_0$ , is plotted by the solid line, and  $D^+D^{*-}$  wave function,  $r\varphi(r)_+$ , by the dashed line.

ordinate space are

$$r\varphi(r)_0 = \left(\frac{\pi}{2}\right)^{1/2} \frac{N_0}{\Lambda^2 - \alpha_0^2} (e^{-\alpha_0 r} - e^{-\Lambda r}), \quad (16)$$

and

$$r\varphi(r)_+ = \left(\frac{\pi}{2}\right)^{1/2} \frac{N_+}{\Lambda^2 - \alpha_+^2} (e^{-\alpha_+ r} - e^{-\Lambda r}), \quad (17)$$

with

$$N_0 = 2\mu_0 \frac{g}{\sqrt{\Lambda}} \left(\frac{c_1}{c_2}\right), \quad N_+ = 2\mu_+ \frac{g}{\sqrt{\Lambda}} \left(\frac{c_1}{c_3}\right). \quad (18)$$

Each of the neutral and the charged  $D\bar{D}^*$  components of the wave function of the bound state with  $\Lambda = 0.5$  GeV and  $m_X = 3.87168$  GeV is shown in Fig. 1. It is also found that the radius of the  $D^+D^{*-}$  component is much smaller than that of  $D^0\bar{D}^{*0}$ . In Fig. 2, we show the wave function of the  $X(3872)$  also for  $m_X = 3.8687$  GeV. One finds that the size of the bound state, especially the size of  $D^0\bar{D}^{*0}$  component, becomes much smaller than that in Fig. 1 though it is still much larger than the usual charmonium, whose rms  $\lesssim 1$  fm [65].

### III. SPECTRUM

In this section, we investigate the transition strength  $S(E)$  of the  $B$  meson weak decay:  $B \rightarrow c\bar{c}K$  and  $c\bar{c}$

TABLE III: Coefficients of the  $X$  wave function.

$\Lambda$	$c_1$	$c_2$	$c_3$	$c_{I=0}$	$c_{I=1}$	$m_X$
0.3	0.227	-0.947	-0.228	-0.831	-0.508	
0.5	0.293	-0.920	-0.259	-0.834	-0.468	3.87168
1.0	0.404	-0.871	-0.280	-0.814	-0.418	
0.5	0.522	-0.727	-0.447	-0.830	-0.198	3.8687

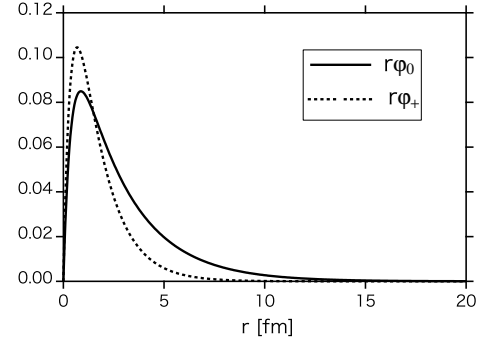


FIG. 2: The  $D\bar{D}^*$  components of the  $X(3872)$  wave function for the  $m_X = 8687$  GeV and  $\Lambda = 0.5$  GeV case. The legend is as for Fig. 1.

to  $X(3872)$  or  $D\bar{D}^*$ . The  $X(3872)$  appears as a bound state in the spectrum. This spectrum does not correspond directly to the observed pion distribution in the  $X(3872) \rightarrow J/\psi\pi^n$  experiments. By looking into the  $D\bar{D}^*$  spectrum, however, one can see that the strength of  $D\bar{D}^*$  gathers around the threshold, and that the peak corresponding to the  $c\bar{c}$  core actually disappears.

In this article, we assume that the observed  $X(3872)$  corresponds to a very shallow bound state. To have such a bound state, the interaction must be attractive but maybe a rather weak one. As mentioned in Sect. II, we have fixed the strength of the  $D\bar{D}^*-c\bar{c}$  coupling,  $g$ , so as to reproduce the observed  $X(3872)$  mass. In such a situation, the  $c\bar{c}$  core state becomes a resonance appearing in the  $D\bar{D}^*$  continuum. Since no sharp resonance is observed experimentally around 3.95 GeV, the width of this resonance should be large. One of the issues in this section is whether such a ‘weak’ attraction can give a resonance with a large decay width.

The  $S(E)$  is normalized so that the production of the  $2^3P_1$   $c\bar{c}$  state by the weak decay is equal to one. The vertex of the weak decay process,  $B \rightarrow c\bar{c} + K$ , and the probability that the  $c\bar{c}$  is in the  $2^3P_1$  configuration are factorized out. We assume that among the  $c\bar{c}$  states produced by the weak decay, the  $2^3P_1$   $c\bar{c}$  state plays a major role to form the  $X(3872)$  and the  $D\bar{D}^*$  spectrum up to around  $E \sim 4$  GeV because the predicted mass of the  $2^3P_1$   $c\bar{c}$  state is 3.95 GeV. Again we use the non-relativistic scheme with the relative  $S$ -wave, because the reduced mass of the system is about 1 GeV and we only consider here up to about 0.1 GeV above the threshold.

Then, the  $S(E)$  is expressed as follows.

$$S(E) = \frac{-1}{\pi} \text{Im} \langle c\bar{c} | G(E) | c\bar{c} \rangle, \quad (19)$$

with the Green’s function;

$$G(E) = \frac{1}{E - \hat{H} + i\varepsilon}. \quad (20)$$

Here,  $E$  represents the energy transfer and  $\hat{H}$  is the full Hamiltonian of the  $c\bar{c}$ -core and  $D\bar{D}^*$  system. The state

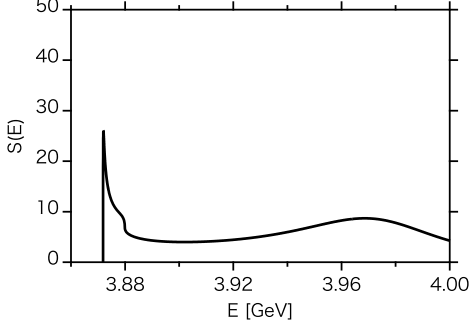


FIG. 3: The transition strength  $S(E)$  with the cutoff  $\Lambda = 0.3$  GeV and the mass of the  $X(3872)$   $m_X = 3.87168$  GeV. The  $c\bar{c} \rightarrow X(3872)$  strength is 0.051. The  $S(E)$  is plotted only for the energy above the  $D^0\bar{D}^{*0}$  threshold. The contribution to the bound state is not plotted here because it does not have a width.

$|c\bar{c}\rangle$  represents the center of mass system of the  $c\bar{c}$  state with the normalization  $\langle c\bar{c}|c\bar{c}\rangle = 1$ . This normalization leads the energy sum rule

$$\int dE S(E) = 1. \quad (21)$$

Using the free Green's functions and the interaction given in Eq. (7), the Green's function is represented as follows.

$$G(E) = G_1^0 + G_1^0 V G_2^0 V G_1^0 + G_1^0 V G_3^0 V G_1^0 + \dots, \quad (22)$$

$$G_1^0(E) = \frac{1}{E - m_{c\bar{c}} + i\varepsilon}, \quad (23)$$

$$G_2^0(E) = \frac{1}{E - m_{D^0} - m_{D^{*0}} - \frac{\hat{p}^2}{2\mu_0} + i\varepsilon}, \quad (24)$$

$$G_3^0(E) = \frac{1}{E - m_{D^+} - m_{D^{*-}} - \frac{\hat{p}^2}{2\mu_+} + i\varepsilon}. \quad (25)$$

The calculated transition strength for the cutoff  $\Lambda = 0.3$  GeV with the mass of the  $X(3872)$   $m_X = 3.87168$  GeV is shown in Fig. 3. The spectrum has a sharp cusp above the  $D^0\bar{D}^{*0}$  threshold. The resonance which corresponds to the  $\chi_{c1}(2P)$  becomes very broad. The bound  $X(3872)$  is not plotted in the figure because it does not have a width in the scheme. If we consider the experimental inaccuracy of the energy and the  $X(3872) \rightarrow J/\psi\pi\pi$  decay width, the bound  $X(3872)$  peak and the threshold cusp will be merged into one single peak, which corresponds to the observed  $X(3872)$  in the  $J/\psi\pi\pi$  spectrum. By integrating  $S(E)$  to the  $D\bar{D}^*$  continuum state, one can obtain the transfer strength from the  $c\bar{c}$ -core to the bound state. In this case, the former is 0.949 while the latter is 0.051. The  $c\bar{c}$  core state of the bare mass of 3.950 GeV becomes a resonance state of  $E = (3.974 - \frac{i}{2}0.067)$  GeV; its peak position is by 24 MeV shifted upward.

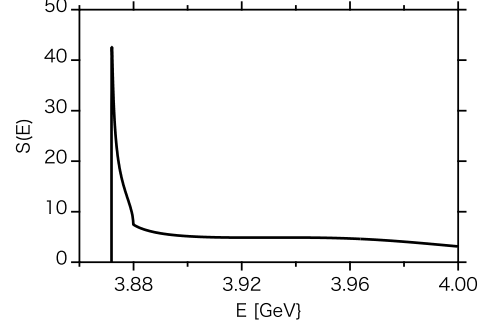


FIG. 4: The transition strength  $S(E)$  with  $\Lambda = 0.5$  GeV and  $m_X = 3.87168$  GeV. The  $c\bar{c} \rightarrow X(3872)$  strength is 0.087. The legend is as for Fig. 3.

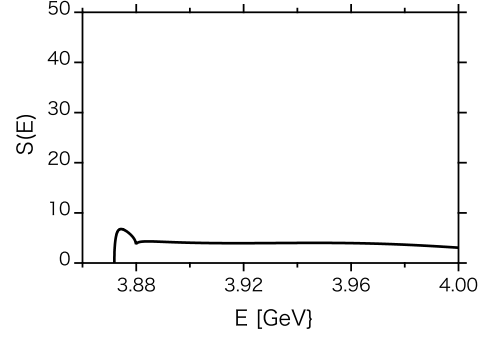


FIG. 5: The transition strength  $S(E)$  with  $\Lambda = 0.5$  GeV and  $m_X = 3.8687$  GeV. The  $c\bar{c} \rightarrow X(3872)$  strength is 0.269. The legend is as for Fig. 3.

In Fig. 4, we show the transition strength for the cutoff  $\Lambda = 0.5$  GeV. The spectrum is almost flat at around  $E = 3.95$  GeV. In the case of this harder cutoff, the  $c\bar{c}$  core state couples to the  $D\bar{D}^*$  continuum of more wider energy range. As a result, the bump around 3.95 GeV found for the  $\Lambda = 0.3$  GeV case disappears. The pole moves to  $E = (3.971 - \frac{i}{2}0.147)$  GeV. The strength from the  $c\bar{c}$ -core to the bound state becomes slightly larger, *i.e.*, 0.087.

Let us now show the effect of the difference in the binding energy. In Fig. 5, we plot the transition strength  $S(E)$  in the case of the cutoff  $\Lambda = 0.5$  GeV and the mass of the  $X(3872)$  is  $m_X = 3.8687$  GeV, *i.e.*, the more deeply binding case. The  $S$ -wave threshold cusp becomes much smaller as the bound state position moves away from the threshold. The transfer strength to the bound state in this case is 0.269, much larger than the previous cases.

Since there is no peak is found around  $E = 3.95$  GeV experimentally, the  $\Lambda = 0.5$  GeV or more is favorable in that sense. This corresponds to the hadron size  $\sim 0.4$  fm, which is a reasonable value. By setting cutoff of this size, the shallow bound state and the large decay width for the  $c\bar{c}$  peak can be realized simultaneously. In the following calculation, we use  $\Lambda = 0.5$  GeV.

#### IV. EFFECT OF THE INTERACTION BETWEEN $D$ AND $\bar{D}^*$

In this section, we introduce the interaction between the  $D$  and  $\bar{D}^*$  mesons. First let us make a rough estimate of the size of the  $D\bar{D}^*$  attraction using the information from the  $B^{(*)}\bar{B}^*$  system. Each of the  $B\bar{B}^*$  and  $B^*\bar{B}^*$  systems has a resonances by about 2~3 MeV above the thresholds,  $Z_b(10610)$  and  $Z_b(10650)$ . Their masses are 10.6072 and 10.6522 GeV, respectively, and their mass difference is 45.0 MeV. The corresponding thresholds,  $B\bar{B}^*$  and  $B^*\bar{B}^*$ , are 10.6048 and 10.6504 GeV, respectively, and the energy difference of these two threshold is 45.6 MeV. The experiment strongly suggests that the two-meson attraction is barely strong enough to make a zero-energy bound state (or somewhat weaker), and that there are almost no mixing between the  $B\bar{B}^*$  and  $B^*\bar{B}^*$  states. The physical origin of the two-meson interaction is probably the light-meson exchange and/or the gluonic interaction. In either case, the strength of the two-meson interaction for the  $D\bar{D}^*$  system is about the same size, or at least a similar size, for the  $B\bar{B}^*$  system, because the bosons exchanging between the light quarks is considered to give the largest contribution. So, as the two-meson interaction between the  $D$  and  $\bar{D}^*$ , we employ the one with the strength which gives a zero-energy bound state for the  $B\bar{B}^*$  systems.

We use the Yamaguchi separable potential [68] for the interaction, namely,

$$\langle MM'(\mathbf{q})|U|MM'(\mathbf{p})\rangle = \frac{-\lambda}{\Lambda^2} \left( \frac{\Lambda^2}{q^2 + \Lambda^2} \right) \left( \frac{\Lambda^2}{p^2 + \Lambda^2} \right), \quad (26)$$

where  $\Lambda$  is the cutoff, and  $\lambda$  is the strength of the interaction. For simplicity, we take the same value for the cutoff  $\Lambda$  in Eq. (26) as that of Eq. (7) in the following calculation. In order to give a zero-energy bound state, the strength should be

$$\lambda = \frac{\Lambda}{\mu_{MM'}} \quad (27)$$

with the reduced mass of the system,  $\mu_{MM'}$ . For  $\Lambda=0.5$  GeV and  $\mu_{B\bar{B}^*}=2.651$  GeV, this strength becomes 0.1886, which we denote  $\lambda_B$  below. As for the  $D\bar{D}^*$ , the required strength to have a zero-energy bound state becomes 0.5712.

First we examine the  $\pi$ -exchange interaction if it can be the origin of the attraction which makes the  $X(3872)$  or  $Z_b$ . The  $\pi$ -meson exchange between the light quarks can be expressed as

$$V_{MM'}^{OPEP} = (\tau \cdot \tau)(\sigma \cdot \sigma)V_0 \left[ -\frac{4\pi}{m_\pi^3} \delta^3(\mathbf{r}) + \frac{\mu^2}{m_\pi^2} \frac{e^{-\mu r}}{m_\pi r} \right] \quad (28)$$

while for the interaction between the light quark and the anti-quark, the factor  $(\tau \cdot \tau)(\sigma \cdot \sigma)$  becomes  $-(\tau \cdot \bar{\tau})(\sigma \cdot \bar{\sigma})$ ,

because the isospin generator for the anti-quark,  $\bar{\tau}$ , corresponds to the transpose of the isospin generator of the quark,  $\tau$ , as  $-\bar{\tau} = \tau$ . The range of the interaction,  $\mu$ , can be obtained by using the vector meson mass,  $m_V$ , and the pseudoscalar meson mass,  $m_P$ , as  $\mu^2 = m_\pi^2 - (m_V - m_P)^2$ . It becomes a complex value for the  $D\bar{D}^*$  system,  $\mu^2 = -(44.5 \text{ MeV})^2$ , whereas  $\mu^2 = (127 \text{ MeV})^2$  for the  $B\bar{B}^*$  system. The strength  $V_0$  is about 1.3 MeV [69].

We assume that  $X(3872)$  is  $J^{PC}=1^{++}$  state. Suppose the state consists of  $D^0\bar{D}^*$ , then the factor  $\langle -(\tau \cdot \bar{\tau})(\sigma \cdot \bar{\sigma}) \rangle$  is +1, whereas it becomes +3 if the state is the isospin 0 state. As for the  $Z_b$ 's, we assume they are  $I(J^P)=1(1^+)$  states. Then the factor becomes also +1 for  $Z_b$  (Table IV). Both of the values corresponds to those obtained from the heavy meson effective lagrangian[69, 70]. As was pointed out in Ref. [69], the Yukawa term and the  $\delta$ -function term in eq. (28) tend to cancel each other. Empirically speaking also, the overall effects of the OPE interaction is expected to be small because the factor  $\sigma \cdot \sigma$  will cause a mixing between the  $B\bar{B}^*$  and  $B^*\bar{B}^*$  states. It has been reported that the OPE interaction (with the tensor term and higher order partial wave states) makes a bound state[70]. There, however, they found that one bound state below the  $B\bar{B}^*$  threshold and one resonance above the  $B^*\bar{B}^*$  threshold rather than two similar resonances. The spin dependence of OPEP seems inconsistent with the  $B^{(*)}\bar{B}^*$  experiments, where the energy difference of the two peaks is almost the same as that of the two thresholds.

Thus, it seems in the  $B^{(*)}\bar{B}^*$  systems, the two-meson interaction is considered to be, at least mostly, spin-independent isospin-independent and is just strong enough to make the almost-zero-energy states. In summary, the  $D\bar{D}^*$  interaction we employ is:

$$\begin{aligned} \langle D^0\bar{D}^{*0}(\mathbf{q})|U|D^0\bar{D}^{*0}(\mathbf{p})\rangle &= \langle D^+D^{*-}(\mathbf{q})|U|D^+D^{*-}(\mathbf{p})\rangle \\ &= \frac{-\lambda}{\Lambda^2} \left( \frac{\Lambda^2}{q^2 + \Lambda^2} \right) \left( \frac{\Lambda^2}{p^2 + \Lambda^2} \right) \end{aligned} \quad (29)$$

with  $\lambda = \lambda_B$ . Though, we look into the effects of the  $D\bar{D}^*$  attraction by changing the value of  $\lambda$  from  $\lambda_B$ .

Let us go back to the Schrödinger equation, which now includes the two-meson interaction,  $U$ :

TABLE IV: The spin-isospin matrix elements of the OPEP by the two-meson states:  $D\bar{D}^*$   $J^{PC} = 1^{++}$  and  $B^{(*)}\bar{B}^*$   $I(J^P) = 1(1^+)$ .

$\langle -(\tau \cdot \bar{\tau})(\sigma \cdot \bar{\sigma}) \rangle$	$D^0\bar{D}^{*0}$	$D^+\bar{D}^{*-}$
$D^0\bar{D}^{*0}$	1	2
$D^+\bar{D}^{*-}$	2	1
$\langle -(\tau \cdot \bar{\tau})(\sigma \cdot \bar{\sigma}) \rangle$	$B^+\bar{B}^{*0}$	$B^{*+}\bar{B}^{*0}$
$B^+\bar{B}^{*0}$	1	2
$B^{*+}\bar{B}^{*0}$	2	1

$$\begin{pmatrix} m_{c\bar{c}} - E & V & V \\ V & m_{D^0} + m_{\bar{D}^{*0}} + \frac{\vec{p}^2}{2\mu_0} + U - E & 0 \\ V & 0 & m_{D^+} + m_{\bar{D}^{*-}} + \frac{\vec{p}^2}{2\mu_+} + U - E \end{pmatrix} \begin{pmatrix} c_1 |c\bar{c}\rangle \\ c_2 |D^0 \bar{D}^{*0}\rangle \\ c_3 |D^+ \bar{D}^{*-}\rangle \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \quad (30)$$

Now the model has one more parameter,  $\lambda$ , which stands for the coupling strength of the interaction between the  $D$  and  $\bar{D}^*$ , in addition to the cutoff  $\Lambda$  and the coupling strength between the  $c\bar{c}$  core and the two-meson state,  $g$ . The  $\lambda = 0$  limit corresponds to the results in Sec. II, where we determined the coupling strength  $g$  so as to reproduce the observed  $X(3872)$  mass without introducing the direct  $D\bar{D}^*$  attraction. We call that value  $g_0$  in the following and use it as a reference.

In order to study the effects of the interaction between the  $D$  and  $\bar{D}^*$ , we change the value of the coupling constant  $\lambda$ . For a positive  $\lambda$ ,  $g$  should be smaller than  $g_0$  in order to reproduce the observed mass of the  $X(3872)$ . Or, equivalently, when  $(g/g_0)^2 < 1$ , one has to take  $\lambda > 0$  to compensate the weakened coupling. At the  $g = 0$  limit, the  $X(3872)$  becomes a pure  $D^0 \bar{D}^{*0}$  hadronic molecular state. There is no charmonium component nor the  $D^+ \bar{D}^{*-}$  component in the  $X(3872)$ . There will be a similar bound state in the  $D^+ \bar{D}^{*-}$  system also, and the  $c\bar{c}$  core becomes a sharp resonance at around 3.95 GeV. We consider the actual situation is in-between of the two  $\lambda = 0$  and  $g = 0$  limits.

In Fig. 6, we show the size of each of the  $c\bar{c}$ , the  $D^0 \bar{D}^{*0}$  and the  $D^+ \bar{D}^{*-}$  components in the  $X(3872)$  wave function in our calculation. For each values of  $(g/g_0)^2$ , we re-adjust the value of  $\lambda$  to fit the mass of the  $X(3872)$  to be 3.87168 GeV. In Fig. 7, we also plot the sizes of each of the isovector and the isoscalar  $D\bar{D}^*$  components. As the interaction between the  $D$  and  $\bar{D}^*$  becomes larger (*i.e.*,  $(g/g_0)^2$  becomes smaller), the isovector  $D\bar{D}^*$  component

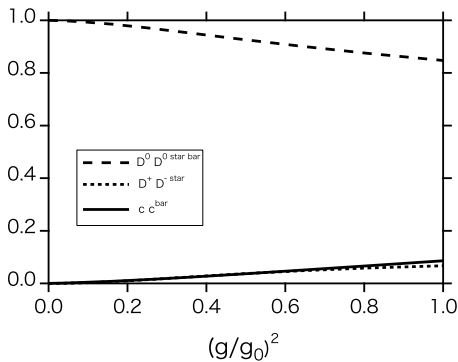


FIG. 6: Probability of each components in  $X(3872)$ . We take the mass of the  $X(3872)$   $m_X = 3.87168$  GeV with the cutoff  $\Lambda = 0.5$  GeV. The solid line shows the size of the  $c\bar{c}$  component in  $X(3872)$ , the dotted line shows that of the  $D^+ \bar{D}^{*-}$  and the dashed line shows that of the  $D^0 \bar{D}^{*0}$ .

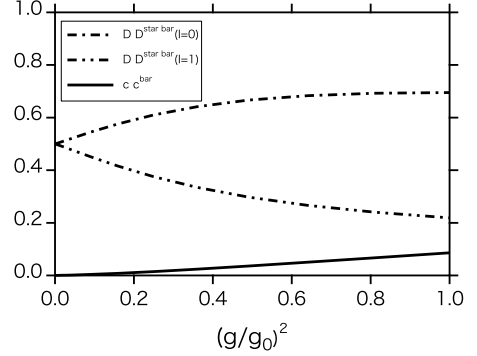


FIG. 7: Probability of each components in  $X(3872)$ . The parameters are the same as those in Fig. 6. The solid line shows the size of the  $c\bar{c}$  component in  $X(3872)$ , the dash double dotted line shows that of the isovector  $D\bar{D}^*$  and the dash dotted line shows that of the isoscalar  $D\bar{D}^*$ .

in the  $X(3872)$  wave function becomes larger while the isoscalar  $D\bar{D}^*$  component reduces to 0.5.

As was mentioned before, experimentally the isovector component seems to be about one forth of the isoscalar component (see eq. (2)). Also, the production process of  $X(3872)$  suggests that there should be a sizable  $c\bar{c}$  component. From Fig. 7, one can find that these requirements are fulfilled when  $(g/g_0)^2$  is close to 1, namely the  $\lambda = 0$  limit.

When the  $D\bar{D}^*$  interaction is switched on, and its strength becomes  $\lambda = \lambda_B$ , the coupling to the  $c\bar{c}$  core

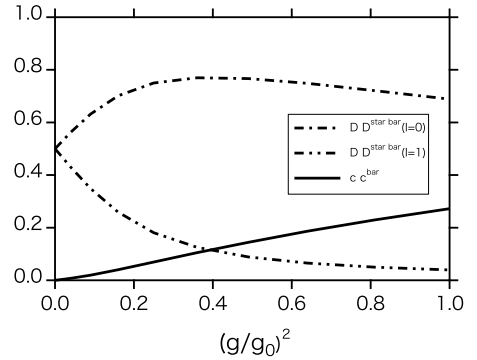


FIG. 8: Probability of each components in  $X(3872)$ . We take the mass of the  $X(3872)$   $m_X = 3.8687$  GeV with the cutoff  $\Lambda = 0.5$  GeV. The legend is as for Fig. 7.



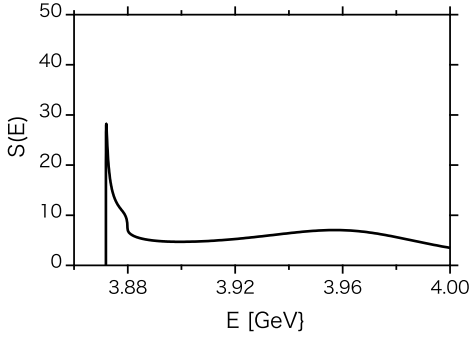


FIG. 9: The calculated transition strength  $S(E)$  with  $\Lambda = 0.5$  GeV and  $m_X = 3.87168$  GeV with  $(g/g_0)^2 = 0.699$ . The  $c\bar{c} \rightarrow X(3872)$  strength is 0.056. The legend is as for Fig. 1.

becomes  $g = 0.0427315$ , which corresponds to  $(g/g_0)^2 = 0.699$ . This point also gives the appropriate size of the isospin symmetry breaking as well as the sizable  $c\bar{c}$  component. There each of the components of the  $X(3872)$  wave function is:

$$\begin{aligned} |X\rangle &= 0.237 |c\bar{c}\rangle - 0.944 |D^0 \bar{D}^{*0}\rangle - 0.0228 |D^+ D^{*-}\rangle \\ &= 0.237 |c\bar{c}\rangle - 0.829 |D\bar{D}^*; I=0\rangle - 0.506 |D\bar{D}^*; I=1\rangle. \end{aligned} \quad (31)$$

This result means that about 6% of the  $X(3872)$  is the charmonium, about 69% is the isoscalar  $D\bar{D}^*$  molecule and 26% is the isovector  $D\bar{D}^*$  molecule. Provided that the rhs of Eq. (2) corresponds faithfully to the ratio of the isovector to the isoscalar  $D\bar{D}^*$  molecular components in the  $X(3872)$  wave function as it is, the state expressed by Eq. (31) is consistent with the experiment. This situation seems to depend on the  $(g/g_0)^2$  value only mildly.

We have also solved the system where the mass  $m_X = 3.8687$  GeV, namely, by about 3 MeV more bound case. The components in such a case are shown in Fig. 8. Here, the  $c\bar{c}$  component is much larger than that of  $m_X = 3.87168$  GeV. The size of the  $c\bar{c}$  core component is sensitive to the value of the binding energy. To make the mass  $m_X = 3.8687$  GeV, the strength becomes  $g = 0.04873$ , which corresponds to  $(g/g_0)^2 = 0.750$ . The core component becomes large in this situation, though the isovector component becomes somewhat smaller.

In Fig. 9, we plot the transition strength  $S(E)$  for the  $\Lambda = 0.5$  GeV and  $m_X = 3.87168$  GeV with  $(g/g_0)^2 = 0.699$  case. Also when the  $D\bar{D}^*$  interaction is introduced, it is found that the strength gathers close to the thresholds. The strength to the  $X(3872)$  is 0.056. The peak around the  $c\bar{c}$  core disappears due to the coupling between the two-meson states and the  $c\bar{c}$  core. It becomes a resonance of  $E = (3.966 - {}^{+0.091}_{-0.091})$  GeV.

Thus, we conclude that in case of the  $X(3872)$ , rather small amount of the interaction is coming from the direct interaction between the  $D$  and  $D^*$  mesons and that the rest of the attraction is coming from the coupling to the  $c\bar{c}$  core state. Then we have the right size of the isospin symmetry breaking as well as a sizable  $c\bar{c}$  component,

both of which are key features to explain the experiments. Also, this picture is consistent with the existence of  $Z_b$  resonances and absence of the charged  $X$ .

## V. CONCLUSIONS

In this work, we have studied the structure of the  $X(3872)$  as well as the transfer strength from the  $c\bar{c}$  core to the  $D\bar{D}^*$  scattering state. The system consists of  $D^0 \bar{D}^{*0}$ ,  $D^+ D^{*-}$ , and the  $2^3P_1$   $c\bar{c}$  core, which stands for the  $\chi_{c1}(2P)$  if observed. We have introduced the direct interaction between the two mesons, which is just as attractive as the one which makes a zero-energy bound state if applied to the  $B^{(*)}\bar{B}^*$  system. Namely, we assume that this two-meson interaction gives the  $Z_b(10610)$  and  $Z_b(10650)$  resonances. This interaction, however, is not strong enough to make a bound state in the  $D\bar{D}^*$  systems alone. In this model, the coupling between the  $c\bar{c}$  core and the  $D\bar{D}^*$  two-meson state is also introduced, which effectively produces the attraction between the  $D$  and  $\bar{D}^*$ . We assume that this coupling provides the rest of the attraction required to make a bound state in the  $D\bar{D}^*$  system,  $X(3872)$ . Both of the interaction and the coupling are assumed to be isospin independent. The isospin symmetry breaking in this model solely comes from the mass difference between the neutral and charged  $D$  and the  $D^*$  mesons.

In the obtained wave function of the  $X(3872)$ , there is about 6% of the  $c\bar{c}$  core component. This size is consistent with a rough estimate from the  $X(3872)$  production rate in the  $p\bar{p}$  collision. As for the  $D\bar{D}^*$  components of the  $X(3872)$  wave function, 69% is isoscalar and 26% is isovector; the ratio of the isovector to the isoscalar  $D\bar{D}^*$  components is also consistent with the experiments of the final  $\pi^2$  to  $\pi^3$  decay ratio. The present work shows that the structure of the  $X(3872)$  is not a simple  $c\bar{c}$  nor a simple  $D^0 \bar{D}^{*0}$  bound state. It is charmonium-hadronic molecule hybrid, which is certainly an exotic state.

Since the  $c\bar{c}$  core cannot couple to the charged  $D\bar{D}^*$  states, such as  $D^+ \bar{D}^{*0}$ , the present picture can explain why there exists no charged partners of the  $X(3872)$ . Also, it can explain why the  $2^3P_1 c\bar{c}$  core, or  $\chi_{c1}(2P)$ , is not found experimentally though it has been predicted by the quark model which gives correct mass spectrum below the open charm threshold; this core couples strongly to the  $D\bar{D}^*$  two-meson state and becomes a resonance with a very broad width.

In order to confirm the present picture of the  $X(3872)$ , we consider that the inclusion of the  $\rho J/\psi$  and  $\omega J/\psi$  channels is important because the  $X(3872)$  is mainly observed in the  $X(3872) \rightarrow \rho J/\psi \rightarrow \pi\pi J/\psi$  and  $X(3872) \rightarrow \omega J/\psi \rightarrow \pi\pi\pi J/\psi$  channels. We are now performing such calculations and the results will be reported soon.

Recently, Belle Collaboration reported the results of the radiative decays of the  $X(3872)$  [71]. They searched the  $X(3872) \rightarrow \psi'\gamma$  in B decays, but no significant signal

has been found. On the other hand, *BABAR* Collaboration has reported that  $\mathcal{B}(X(3872) \rightarrow \psi' \gamma)$  is almost 3 times that of  $\mathcal{B}(X(3872) \rightarrow J/\psi \gamma)$  [12]. To make the situation clear, it is useful to calculate the radiative decays of the  $X(3872)$  in the present model including the charmonium structure. It is left as the future study.

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